## What is the parameter being estimated?

<table>
<thead>
<tr>
<th><strong>Population Proportion, ( P )</strong></th>
<th><strong>Population Mean, ( \mu ) (( \sigma ) unknown)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Verify</strong> the following before proceeding with constructing the confidence interval:</td>
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</tr>
<tr>
<td>1. Data obtained via a random sample</td>
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</tr>
<tr>
<td>2. ( n \leq 0.05N ), and</td>
<td>2. Sample size is small relative to the population: ( n \leq 0.05N )</td>
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<tr>
<td>3. ( n\hat{p}(1 - \hat{p}) \geq 10 ) (or ( n\hat{p} \geq 5 ) and ( n(1 - \hat{p}) \geq 5 ))</td>
<td>3. The sample comes from a population that is normally distributed or the sample size is large (( n \geq 30 )).</td>
</tr>
<tr>
<td>If you don’t know whether or not the population is normal, use a <strong>Normal Probability Plot</strong>, to confirm that it is reasonable to assume the population is normal</td>
<td></td>
</tr>
<tr>
<td>4. There are no outliers: (Use a <strong>Boxplot</strong>, to make sure that there are no outliers.)</td>
<td></td>
</tr>
</tbody>
</table>

The \((1 - \alpha)100\%\) confidence interval (Z-interval) for the population proportion \( P \) would be:

- **Lower Bound:** \( \hat{p} - \frac{z_{\alpha}}{2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \)
- **Upper Bound:** \( \hat{p} + \frac{z_{\alpha}}{2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \)

**Example:** A drug company has a popular drug that is effective for relieving cold symptoms. However, they have also found that, in some cases, insomnia is a side effect of this drug. They conduct a survey of 500 adults who have taken their drug and find that 30 experienced insomnia while on the drug. Construct the 96% confidence interval for the proportion of all adults who are taking their drug and are experiencing insomnia.

The \((1 - \alpha)100\%\) confidence interval (t-interval) for the population mean \( \mu \) would be

- **Lower Bound:** \( \bar{x} - t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \)
- **Upper Bound:** \( \bar{x} + t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \)

where \( t_{\alpha/2} \) is the critical value with \( n - 1 \) degrees of freedom.

**Example:** A national bookstore chain is considering selling electronic copies of popular books. They conduct a survey of 100 of their adult customers and ask them “During the past year, how many ebooks did you purchase?” The results of the survey indicated that \( \bar{x} = 15.2 \) ebooks and \( s = 8.2 \) ebooks. Construct a 99% confidence interval for the mean number of ebooks purchased by their customers. (Assume no outliers.)
Solution:
First verify the conditions necessary to construct a confidence interval are met:

- Is \( n \leq 0.5 \, N \)? The sample of 500 adults (\( n \)), is certainly less than 5% of all adults taking their drug (\( N \)). Thus, this requirement is met.

- Is \( n \hat{p}(1-\hat{p}) \geq 10 \)? \( \hat{p} = \frac{30}{500} = 0.06 \)
  \[ n \hat{p}(1-\hat{p}) = (500)(0.06)(1 - 0.06) = 28.2 \geq 10 \]
  Therefore, the sampling distribution of \( \hat{p} \) is approximately normal.

We have satisfied the requirements; therefore, we can now construct the 96% confidence interval.

- Since we are using a 96% confidence level, \( \alpha = 0.04 \)
- \( z_{\alpha/2} = z_{0.04/2} = z_{0.02} = 2.054 \) (from the calculator)

- Therefore, the 96% confidence interval is:
  \[ \hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \]
  \[ = 0.06 \pm (2.054) \cdot \sqrt{\frac{0.06(1-0.06)}{500}} \]
  or (0.0382, 0.0818)

Conclusion: We are 96% confident that the proportion of all users of this cold medicine who experience insomnia while on the drug is between 0.0382 and 0.0818.

Solution:
First verify the conditions necessary to construct a confidence interval are met:

- Is \( n \leq 0.5 \, N \)? One hundred customers of a national bookstore chain is certainly less than 5% of all of their customers and no outliers.

- \( n \geq 30 \) \( n = 100 \geq 30 \) Thus this requirement is met. (We need this requirement to be met because we don’t know whether the population is normally distributed or not.)

We will construct the 99% confidence interval.

- Since we are using the 99% confidence interval, \( \alpha = 0.01 \) and because \( n = 100 \), there are \( 100 - 1 = 99 \) degrees of freedom. Since 99 is not in the table we will use 100 degrees of freedom.

- Thus, \( t_{\alpha} = t_{0.01} = 2.364 \) (from table VI)

- Therefore the 99% confidence interval is:
  \[ \bar{x} \pm t_{\alpha} \cdot \frac{s}{\sqrt{n}} \]
  \[ = 15.2 \pm (2.364) \cdot \frac{8.2}{\sqrt{100}} \]
  or (13.26, 17.14)

Conclusion: We are 99% confident that the mean number of ebooks purchased by the chain’s customers is between 13.26 ebooks to 17.14 ebooks.
<table>
<thead>
<tr>
<th>Solution using the TI-84 function</th>
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<tbody>
<tr>
<td>Press STAT, then TESTS, then 1 – PropZINT</td>
<td>Press STAT, then TESTS, then TInterval</td>
</tr>
<tr>
<td>(0.03819, 0.08181)</td>
<td>(13.046, 17.354)</td>
</tr>
</tbody>
</table>