# Using Double-Angle Formulas

**Objective:** To apply the double-angle formulas for the sine, cosine, and tangent functions

- The double-angle formulas allow us to change the angle being plugged into certain trigonometric functions.

- You need to memorize the following double-angle formulas:
  
  \[
  \sin 2x = 2 \sin x \cos x \\
  \cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x \\
  \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}
  \]

- Notice that there are three versions of the double-angle formula for cosine. Also note that the angle on the left-hand side of the equation is always \(2x\) while the angles on the right have changed to \(x\).

## Example

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| If \(\cos x = \frac{3}{5}\) and \(x\) is in Quadrant I, find \(\sin 2x, \cos 2x,\) and \(\tan 2x\). | \[
\cos 2x = 2 \cos^2 x - 1 \\
= 2 \left(\frac{3}{5}\right)^2 - 1 = 2 \left(\frac{9}{25}\right) - 1 \\
= \frac{18}{25} - 1 = -\frac{7}{25}
\] | - To find \(\cos 2x\), use the version of the double-angle formula only in terms of \(\cos x\), since that is given. |
| | \[
\sin 2x = 2 \sin x \cos x \\
= 2 \left(\frac{4}{5}\right) \left(\frac{3}{5}\right) = \frac{24}{25}
\] | - To find \(\sin 2x\), we need to know \(\sin x\) as well. So use a right triangle or the Pythagorean Identity \(\sin^2 x + \cos^2 x = 1\) Note that \(\sin x\) must be positive because \(x\) is in Quadrant I. |
| | \[
\tan 2x = \frac{\sin 2x}{\cos 2x} \\
= \frac{-\frac{7}{25}}{\frac{24}{25}} = -\frac{7}{25} \cdot \frac{25}{24} = -\frac{7}{24}
\] | - To find \(\tan 2x\), use the fundamental identity that \(\tan 2x = \frac{\sin 2x}{\cos 2x}\). |
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| Simplify the expression \[
\frac{2 \tan 20^\circ}{1 - \tan^2 20^\circ}
\] | \[
\frac{2 \tan x}{1 - \tan^2 x} = \frac{2 \tan 20^\circ}{1 - \tan^2 20^\circ}
\] \[
= \tan 2(20^\circ)
\] \[
= \tan 40^\circ
\] | • This looks like the right side of the double-angle formula for tangent, with \(x\) replaced with 20°. This means that \(x\) can be replaced with 20° on the left side as well.  
• If we know the exact value of the simplified expression from the unit circle, we can evaluate. |

### Exercises:

1) If \(\sin x = \frac{12}{13}\) and \(x\) is in Quadrant II, find \(\sin 2x\), \(\cos 2x\), and \(\tan 2x\).

2) If \(\cos x = \frac{\sqrt{3}}{3}\) and \(x\) is in Quadrant IV, find \(\sin 2x\), \(\cos 2x\), and \(\tan 2x\).

3) If \(\tan x = 4\) and \(x\) is in Quadrant III, find \(\sin 2x\), \(\cos 2x\), and \(\tan 2x\).

4) Simplify and find the exact value, if possible, of the following expression: \(1 - 2 \sin^2 \frac{\pi}{12}\)

5) Simplify and find the exact value, if possible, of the following expression: \(2 \sin 22.5^\circ \cos 22.5^\circ\)

6) Simplify and find the exact value, if possible, of the following expression: \[
\frac{2 \tan \frac{5\pi}{8}}{1 - \tan^2 \frac{5\pi}{8}}
\]
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Solutions:

1) \( \sin 2x = 2 \sin x \cos x = 2 \left( \frac{12}{13} \right) \left( -\frac{5}{13} \right) = -\frac{120}{169} \)

\[ \cos 2x = \cos^2 x - \sin^2 x = -\left( \frac{5}{13} \right)^2 - \left( \frac{12}{13} \right)^2 = \frac{25}{169} - \frac{144}{169} = -\frac{119}{169} \]

\[ \tan 2x = \frac{\sin 2x}{\cos 2x} = \frac{-120/169}{-119/169} = -\frac{120}{169} \cdot \frac{169}{119} = \frac{120}{119} \]

2) \( \sin 2x = -\frac{2\sqrt{18}}{9} \quad \cos 2x = -\frac{1}{3} \quad \tan 2x = \frac{2\sqrt{18}}{3} \)

3) \( \sin 2x = \frac{8}{17} \quad \cos 2x = -\frac{15}{17} \quad \tan 2x = -\frac{18}{15} \)

4) \( 1 - 2 \sin^2 \frac{\pi}{12} = \cos 2 \left( \frac{\pi}{12} \right) = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \)

5) \( \frac{\sqrt{2}}{2} \)

6) 1