Quadratic Models

A mathematical model in two variables is an equation that is used to show the relationship between two quantities. It may be used to describe the behavior of a real-life situation. A quadratic model is a model of the form \( y = ax^2 + bx + c \), where \( a, b \) and \( c \) are real numbers with \( a \neq 0 \). We can use models to determine or predict what happens at a certain instant or when something might occur. See the example below.

**Example:** A ball was thrown upward at an initial velocity of 96 feet per second from a height of 6 feet above the ground. The height, \( h \), of the ball after \( t \) seconds can be given by \( h = -16t^2 + 96t + 6 \).

a) Find the height of the ball 3 seconds after it is thrown?

Solution: We need to find the height for \( t = 3 \). Replace \( t \) with 3 and simplify.

\[
\begin{align*}
h &= -16t^2 + 96t + 6 \\
&= -16(3)^2 + 96(3) + 6 \\
&= -144 + 288 + 6 \\
&= 150
\end{align*}
\]

The ball will be 150 feet above the ground 3 seconds after it is thrown.

b) When will the ball be 86 feet above the ground?

Solution: We need to find \( t \) when \( h = 86 \). Replace \( h \) with 86 and solve.

\[
\begin{align*}
h &= -16t^2 + 96t + 6 \\
-16t^2 + 96t + 6 &= 86 \\
-16t^2 + 96t - 80 &= 0 \\
t^2 - 6t + 5 &= 0 \\
(t - 1)(t - 5) &= 0 \\
t &= 1 \text{ or } t &= 5
\end{align*}
\]

The ball will be 86 feet above the ground in 1 second and 5 seconds after it is thrown.

c) When will the ball reach the ground?

Solution: When the ball is on the ground, it’s height above the ground is 0 feet. We need to find \( t \) when \( h = 0 \).
0 = −16t^2 + 96t + 6

−16t^2 + 96t + 6 = 0

8t^2 − 48t − 3 = 0

Divide both sides the common factor of −2. Solve using the quadratic formula with

\[ a = 8, \quad b = −48, \quad c = −3 \]

\[
t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
t = \frac{−(−48) \pm \sqrt{(−48)^2 − 4(8)(−3)}}{2(8)}
\]

\[
t = \frac{48 \pm \sqrt{2304 + 96}}{16}
\]

\[
t = \frac{48 \pm \sqrt{2400}}{16}
\]

\[
t = \frac{48 + \sqrt{2400}}{16} \approx 6.1 \quad \text{or} \quad t = \frac{48 - \sqrt{2400}}{16} \approx −0.1
\]

The ball will reach the ground in approximately 6 seconds after it is thrown.

**Practice Exercises:**

1) A ball is dropped from the top of the Eiffel Tower which is 990 feet tall. The height of the ball \( t \) seconds after it is dropped is given by \( h = −16t^2 + 990 \).

   a) How high will the ball be 5 seconds after it is dropped?
   b) When will the ball be 846 feet above the ground?
   c) When will the ball reach the ground?

2) The function defined by \( D(t) = 13t^2 − 73t \) gives the distance in feet that a car going approximately 50 mph will skid in \( t \) seconds.

   a) How far will the car skid in 6 seconds?
   b) Find the time it will take the car to skid 445 feet.

**Answers:**

1a) 590 feet  b) 3 seconds  c) 7.03 seconds
2a) 30 feet  b) 9.3 seconds