EER#21- Graph parabolas and circles whose equations are given in general form by completing the square.

Circles
A circle is a set of points that are equidistant from a fixed point. The distance is called the radius of the circle and the fixed point is called the center.

The standard form of the equation of a circle is

\[
(x-h)^2 + (y-k)^2 = r^2 \\
(h,k) = \text{Center and } r = \text{Radius}
\]

We can find the equation of a circle given the center and the radius or given the center and a point on the circle. Let’s look at a couple of examples.

1. Find the equation of a circle with a center (-3,4) and a radius of 6.
   To find the equation, all we have to do is enter the center for h and k, and the radius for r.
   \[
   (x-(-3))^2 + (y-4)^2 = 6^2 \\
   (x+3)^2 + (y-4)^2 = 36
   \]

2. Find the equation of a circle with center (2,-7) and passing through the point (-2,-4).
   We can plug in the center for h and k, and the point for x and y to allow us to solve for the radius.
   \[
   (x-2)^2 + (y+7)^2 = r^2 \\
   (-2-2)^2 + [-4-(-7)]^2 = r^2 \\
   (-4)^2 + (-4+7)^2 = r^2 \\
   (-4)^2 + (3)^2 = r^2 \\
   16 + 9 = r^2 \\
   25 = r^2 \\
   \sqrt{25} = \sqrt{r^2} \\
   5 = r
   \]
   The radius is 5, so the standard form for the equation of this circle is
   \[
   (x-2)^2 + (y+7)^2 = 5^2 \\
   (x-2)^2 + (y+7)^2 = 25
   \]
You can also use this information to help you graph circles. As long as you know the center and the radius, you can graph a circle by hand. *(It is much easier to graph a circle by hand than on the calculator.)*

**Graphing Circles**

The only thing you need to do in order to graph a circle is find the center and the radius. Graph the center point, then move every direction \( r \) units to graph the points of the circle. Let’s look at an example.

**Graph:** \((x+8)^2 + (y-2)^2 = 16\)

Looking at the equation of the circle, we can see that it’s in standard form. Thus, we can look at it and see that the center is \((-8,2)\) and the radius is 4. So, to graph this circle, plot the center point and move four units in every direction to get the points of the circle. The graph is shown below.

If the equation is not given to you in standard form, then you have to use a process called **Completing The Square** to put it in standard form. All equations of circles are either in standard or general form.

**General Form**

\[ x^2 + y^2 + cx + dy + e = 0 \]

(where c, d and e are real numbers)

If an equation is given to you in general form, you cannot just look at it and determine the center and radius. Thus, you have to put it in standard form first. To go from general to standard form, you **complete the square**.

**Completing the Square**

1. Take the constant \((e)\) to one side of the equation.
2. Divide both sides by the \(x^2/y^2\) coefficient (if there is one).
   (Note-If there is a number in front of \(x^2\) & \(y^2\), it will be the same.)
3. Take \(\frac{1}{2}\) of the \(x\) and \(y\)-coefficients and square it.
4. Add this result to both sides of the equation.
5. Factor.

Once you have factored, the equation will be in standard form. Let’s look at a couple of examples.

1. Find the center and radius of the equation: \( x^2 – 6x + y^2 + 4y – 3 = 0 \)
   This equation is given in general form, so we have to complete the square and right it in standard form. Follow the steps given above.

\[
x^2 – 6x + y^2 + 4y = 3
\]

Take the constant to the other side of the equation.

We don’t have a leading coefficient, so we take \( \frac{1}{2} \) of the x and y-coefficients then square it and add this number to both sides of the equation.

Thus, add 9 and 4 to both sides of the equation, and you get:

\[
x^2 – 6x + y^2 + 4y = 3
x^2 – 6x + 9 + y^2 + 4y + 4 = 3 + 9 + 4
\]

Now we can factor the left-hand side.

\[(x- 3)^2 + (y+2)^2 = 16\]

Thus, the center is (3,-2) and the radius is 4.

2. \( 2x^2 + 2y^2 – 6x + 10y = 1 \)
   This equation is not in standard form, so we must complete the square. Since the constant is already on the right, we can start by dividing both sides of the equation by the leading coefficient (which is 2).

\[
\frac{2x^2 + 2y^2 – 6x + 10y}{2} = \frac{1}{2}
\]

\[
x^2 + y^2 – 3x + 5y = \frac{1}{2}
\]

\[
x^2 – 3x + y^2 + 5y = \frac{1}{2}
\]

\[
x^2 – 3x + \frac{9}{4} + y^2 + 5y + \frac{25}{4} = \frac{1}{2} + \frac{9}{4} + \frac{25}{4}
\]

\[
\left(x – \frac{3}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{36}{4}
\]
Parabolas

You have worked with parabolas earlier in the textbook, but now we are looking at parabolas that are not functions as well. The parabolas you have looked at so far are oriented vertically (facing up or down). Now, you will look at parabolas oriented horizontally (facing left or right).

The definition of a parabola is the set of all points equidistant from a fixed point and a fixed line. The fixed point is called the focus and the fixed line is called the directrix.

In this course, you will only be required to write the equation in standard form by completing the square, find the vertex, find the orientation of the graph, find the axis of symmetry, and graph the equation.

Below are the standard forms of the equation of a parabola, along with examples of graphs showing their axes of symmetry. All equations solved for $y$ will either open up or down. All equations solved for $x$ will open to the left or right.
Let's look at a couple of examples.

Write the equation in standard form, find the vertex, axis of symmetry and graph.

1. \((x – 2)^2 = 8(y – 1)\)

For this problem, we need to rewrite it in standard form, by solving for \(y\) (the non-squared term).

\[(x – 2)^2 = 8(y – 1)\]
\[(x – 2)^2 = 8y – 8\]
\[(x – 2)^2 + 8 = 8y\]
\[\frac{(x – 2)^2 + 8}{8} = \frac{8y}{8}\]
\[\frac{1}{8} (x – 2)^2 + 1 = y\]

So, the vertex is (2,1). The axis of symmetry is x=2. The graph is facing upward. Below is the graph.
1. \( y^2 - 2y - x - 5 = 0 \)

For this problem, we need to rewrite it in standard form, by solving for \( x \) (the non-squared term).

\[
x = y^2 - 2y - 5
\]

The right hand side of the equation cannot be factored, so we must complete the square on \( y \) to get a perfect square. Below are the steps.

**Completing the Square**
1. Take the constant to one side of the equation.
2. Divide both sides by the \( x^2 \) or \( y^2 \) coefficient (if there is one).
3. Take \( \frac{1}{2} \) of the \( x \) or \( y \)-coefficient and square it.
4. Add this result to both sides of the equation.
5. Factor.

Once you have factored, the equation can be written in standard form.

\[
x = y^2 - 2y - 5
\]
\[
x + 5 = y^2 - 2y
\]
\[
x + 5 + 1 = y^2 - 2y + 1
\]
\[
x + 6 = (y - 1)^2
\]
\[
x = (y - 1)^2 - 6
\]

Now that the equation is in standard form we can find the vertex. Since this equation is solved for \( x \), the vertex is \((-6,1)\). The vertex is facing the right. The axis of symmetry is \( y = 1 \). Below is the graph.