Math 1111  Functions

**Headlines**
- Function – relation where each x is paired with only one y
- Domain – set of all x values in a relation
- Range – set of all y values in a relation
- Four Ways to Represent a Function: Verbal (sentence), Algebraic (formula), Visual (graph), and Numeric (table).
- Dependent (output) and Independent (input) variables
- Evaluating functions
- Domain and Range from Table, Equation and Graph
- The Difference Quotient of a Function

Here is a hilarious definition in You Tube:  [http://www.youtube.com/watch?v=DodGaoJi25E](http://www.youtube.com/watch?v=DodGaoJi25E)

**Examples:**

<table>
<thead>
<tr>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>g(x) = 10 + x; 0 ≤ x ≤ 100</td>
<td>c(x) = 59.99; 0 &lt; x ≤ 500 59.99 + 0.05 (x – 500); x ≥ 500</td>
<td>f(x) = x^2 - 2</td>
</tr>
</tbody>
</table>

- **Is g(x) a function? Explain**
- **Is c(x) a function? Explain**
- **Is f(x) a function? Why?**

* **Definition of a Function:** A __________________________ is a rule that assigns to each element x in a set A exactly one element, called __________, in a set B.

In this section, we will look at algebraic form of a Function. We will see how to evaluate a function, find a domain of a function and find the difference quotient of a function.

* **Four Ways to Represent a Function:** Give an example of each type.

<table>
<thead>
<tr>
<th>Verbal</th>
<th>Algebraic</th>
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<table>
<thead>
<tr>
<th>Visual</th>
<th>Numerical</th>
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</table>
Examples:

1. For the relation \{(3,4), (3,5), (4,4), (4,5)\}, find the domain and the range. Is this relation a function (Is y a function of x)?

2. For the relation \{(3,4), (4,5), (5,4), (6,5)\}, find the domain and the range. Is this relation a function (is y a function of x)?

3. Suppose \( y = 3x + 7 \). Is y a function of x? Why?

**EXAMPLES:** Evaluate the function at the indicated values.

<table>
<thead>
<tr>
<th>(A) ( f(x) = x + 5 )</th>
<th>(B) ( f(x) = -2x^2 + x - 4 )</th>
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</thead>
<tbody>
<tr>
<td>( f(0) = )</td>
<td>( f(0) = )</td>
</tr>
<tr>
<td>( f(-a) = )</td>
<td>( f(-1) = )</td>
</tr>
<tr>
<td>( f(a + h) = )</td>
<td>( f(a) = )</td>
</tr>
<tr>
<td>(C) ( f(x) = \sqrt{x - 4} )</td>
<td>(D) ( f(x) = \frac{x^2 + 1}{x - 1} )</td>
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</tr>
<tr>
<td>( f(5) = )</td>
<td>( f(1) = )</td>
</tr>
</tbody>
</table>
\[
\begin{align*}
\text{(E) } f(x) &= \begin{cases} 
  x + 1 & \text{if } x < -2 \\
  |x - 5| & \text{if } x \geq -2 
\end{cases} \\
\text{(F) } f(x) &= \begin{cases} 
  -3 & \text{if } x \leq 1 \\
  \frac{x}{x+2} & \text{if } x > 1 
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\text{f(0) = } & \\
\text{f(-3) = } & \\
\text{f(2) = } & \\
\end{align*}
\]

* **The Domain of a Function:** The ______________ of the function is set of all real numbers for which the function (algebraic expression) is defined as a real number.

**EXAMPLES:** Find the domain of the function.

<table>
<thead>
<tr>
<th>(A) ( f(x) = \sqrt{x} - 4 )</th>
<th>(B) ( f(x) = \frac{x^2 + 1}{x - 1} )</th>
</tr>
</thead>
</table>
| (C) \( f(x) = \frac{x - 3}{\sqrt{x} + 2} \) | (D) \( f(x) = \begin{cases} 
  -3 & \text{if } x \leq 1 \\
  \frac{x}{x+2} & \text{if } x > 1 
\end{cases} \) |

* **The Difference Quotient of a Function:**

**EXAMPLES:** Find the difference quotient of the function.

(A) \( f(x) = x + 5 \)

(B) \( f(x) = \frac{x}{x - 1} \)

Visit [http://plus.maths.org/content/101-uses-quadratic-equation-part-ii](http://plus.maths.org/content/101-uses-quadratic-equation-part-ii) for applications of quadratic functions.


**Graphs of Functions**

**Headlines:**
* Graphing Function by Plotting Points:
* Graphing Function with a Graphing Calculator
* Graphing Piecewise Defined Functions
* Vertical Line Test
* Equations that define Functions

**EXAMPLE:** Sketch the graph of the function by first making a table of values.

(A) \( f(x) = x + 2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>Calculation: ( f(x) = x + 2 )</th>
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(B) \( f(x) = x^2 - 1 \)

<table>
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<th>( x )</th>
<th>( f(x) )</th>
<th>Calculation: ( f(x) = x^2 - 1 )</th>
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**EXAMPLE 2:** Sketch the graph of the piecewise defined function.

(A) \( f(x) = \begin{cases} 
0 & \text{if } x < -2 \\
1 & \text{if } x \geq -2
\end{cases} \)

![Graph of f(x) = x^2 - 1](image1)

![Graph of piecewise defined function](image2)
Equations that define Functions:

When an equation involving x and y is also a function, the equation renamed \( f(x) \) to indicate that “y is a function of x”.

Examples:

1. Does \( x^2 + y^2 = 16 \) describe a function (Is y a function of x)?

2. Suppose \( y = 3x + 7 \). Is y a function of x?

Getting Information from the Graph of a Function

Headlines:

* Definition of Increasing, Decreasing, and Constant Functions:
* Domain and Range
* Local maximum and Minimum Values of a Function

**EXAMPLE 1:** The graph of a function is given. Determine the intervals on which the function is (a) increasing, (b) decreasing, and (c) constant.

<table>
<thead>
<tr>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
</tr>
</thead>
</table>

(B)

\[
f(x) = \begin{cases} 
  x + 1 & \text{if } x < -2 \\
  x + 2 & \text{if } x \geq -2 
\end{cases}
\]
* Local Maxima and Minima of a Function:

- The function value $f(a)$ is a \underline{local maximum} of $f$ if $f(a) \geq f(x)$ when $x$ is near $a$.

  (This means that $f(a) \geq f(x)$ for all $x$ in some open interval containing $a$.)

  In this case we say that $f$ has a \underline{local maximum} at $x = a$.

- The function value $f(a)$ is a \underline{local minimum} of $f$ if $f(a) \leq f(x)$ when $x$ is near $a$.

  (This means that $f(a) \leq f(x)$ for all $x$ in some open interval containing $a$.)

  In this case we say that $f$ has a \underline{local minimum} at $x = a$.

**EXAMPLE 2:** The graph of a function is given. Find all the local maximum and minimum values of the function and the values at which each occurs.
Average Rate of Change of a Function

Headlines:

* The Average Rate of Change of a Function:
* Linear Functions – Constant rate of Change

The **average rate of change** of the function \( y = f(x) \) between \( x = a \) and \( x = b \) is

\[
\text{average rate of change} = \frac{f(b) - f(a)}{b - a}
\]

The average of change is the slope of the _____________________________ between \( x = a \) and \( x = b \) on the graph of \( f \), that is, the line passes through \((a, f(a))\) and \((b, f(b))\).

**Example 4:** Find the average rate of change of the function from \( x_1 \) to \( x_2 \).

| (A) | (B) \( f(x) = \sqrt{x + 2} \) from \( x_1 = 7 \) to \( x_2 = 14 \) |
| (C) | (D) \( f(x) = x^2 - x + 4 \) from \( x_1 = 2 \) to \( x_2 = 5 \) |