College Algebra

Answers to Final Exam Review Problems

Equations

1. \{-1+\sqrt{2}, -1-\sqrt{2}\}
2. \{-2\}
3. \{\frac{1}{3}, -\frac{11}{9}\}
4. \{-3,3,-i\sqrt{3},i\sqrt{3}\}
5. \{-\frac{5}{2}, 3\}
6. \left\{\frac{-4\pm\sqrt{30}}{2}, \frac{-4\pm\sqrt{30}}{2}\right\}
7. \{-4, 2\}
8. \{-2, \sqrt{3}, -\sqrt{3}\}
9. \{5\}
10. x \approx 0.972

Inequalities-- answers are given in interval notation

1. \((-1, \frac{9}{5})\)
2. \((\infty, -2] \cup [0, 2]\)
3. \([2, \infty) \cup \{-3\}\)
4. \((1, 4)\)

Function Topics

1. An example of two ordered pairs which satisfy this equation and show that it is not a function are: \((3, 2)\) and \((3, -2)\). There are many other correct examples.
2. Domain: \(\{x| x \neq \pm 3\}\)
3. Domain: \((\infty, 9]\)
4. Range: \([5, \infty)\)
5. a) \(y = x^3\), or any linear function with non-zero slope  
   b) \(y = e^x\) or \(y = x^3\)  
   c) \(y = x^3\)  
   d) \(y = x^2\) or \(y = |x|\)

6. a) x-axis  
   b) origin  
   c) y-axis  
   d) none  
   e) x-axis, y-axis, origin

7. a) \(y = -\sqrt{x}\)  
   b) \(y = \sqrt{-x}\)  
   c) \(y = \sqrt{x - 2}\)  
   d) \(y = \sqrt{x} + 3\)

8. \(y = (x + 4)^3 - 2\)

9. \(f^{-1}(x) = \frac{3x}{2-x}\)

10. \(\sqrt{13}\)

11. \(9x - 34\)

12. Average rate of change = \(-\frac{1}{5}\)

13. Graph

**Exponents and Logarithms**

1. \(e^5 = x\)

2. \(\log .001 = -3\)

3. \(x \approx 0.0778\)

4. \(x = \frac{3}{32}\)
5. \( x = 25 \ ( x = -4 \text{ is not in the domain}) \)

6. \( x = 5 \)

7. About 22 years

8. a) Domain: \((-4, \infty)\); Range: \((-\infty, \infty)\)
   b) \( f^{-1}(x) = 4^{x+2} - 4 \)
   c) Domain: \((-\infty, \infty)\); Range: \((-4, \infty)\)
   d) \((-1, \infty)\)

9. a) \( A(t) = 8000e^{-0.09589t} \)
   b) About 7.23 hours
   c) About 3066

Analyzing Functions

1. a) Degree = 6 b) Leading Coefficient = 5
   c) As \( x \to \infty, y \to \infty, \text{ and as } x \to -\infty, y \to \infty. \)
   d) The zeros are \(-4, -\frac{2}{5}, \text{ and } 1\)
   e) \(-4\) has multiplicity 2, \(-\frac{2}{5}\) has multiplicity 1, and 1 has multiplicity 3
   f) \((0, -32)\) g) Graph

2. a) There is a vertical translation 4 units up, and a horizontal translation 3 units to the left.
   b) The graph is reflected in the x-axis.
   c) Domain: \([-3, \infty)\) d) Range: \((-\infty, 4]\) e) \((13, 0)\)
   f) \((0, 4-\sqrt{3})\) g) Graph
3. a) Quadratic function, opening down  
   b) Domain: \((-\infty, \infty)\)  
   c) \(V (3, 4)\)  
   d) \(x = 3\)  
   e) Graph  
   f) Range: \((-\infty, 4]\)  
   g) Not one-to-one  
   h) Set equation equal to zero. Solve the quadratic equation by any method. Taking the square root of both sides is the easiest method. 
   Answers are \((3 + \sqrt{2}, 0), (3 - \sqrt{2}, 0)\).  
   i) \((0, -14)\)  
   j) As \(x \to \infty, g(x) \to -\infty\), and as \(x \to -\infty, g(x) \to -\infty\).  
   k) Increasing in the interval: \((-\infty, 3)\)  
   l) \(g(x)\) is greater than 0 for those \(x\)'s in the interval between the two zeros, so, in the open interval \((3 - \sqrt{2}, 3 + \sqrt{2})\)

4. a) Domain: \((-\infty, -\frac{3}{2}) \cup (-\frac{3}{2}, \infty)\)  
   b) Rational function  
   c) \((\frac{1}{5}, 0)\)  
   d) \((0, -\frac{1}{3})\)  
   e) \(VA: x = -\frac{3}{2}; HA: y = \frac{5}{2}\)  
   f) \(f(4) = \frac{19}{11}\)  
   g) \(x = -\frac{13}{3}\)  
   h) Graph  
   i) As \(x \to \infty, f(x) \to \frac{5}{2}\); as \(x \to -\infty, f(x) \to \frac{5}{2}\)  
   j) Range: \(\{y | y \neq \frac{5}{2}\}\)  
   k) Yes, one-to-one.

5. a) Exponential function  
   b) Domain: \((-\infty, \infty)\)  
   c) Asymptote: \(y = -1\) (since the graph was vertically translated one unit down)  
   d) \((-1, 0)\)  
   e) \((0, 1)\)  
   f) Graph  
   g) Range: \((-1, \infty)\)  
   h) As \(x \to \infty, h(x) \to \infty\), and as \(x \to -\infty, h(x) \to -1\).  
   i) Yes, this function is one-to-one.  
   j) \(h(x)\) is less than 0 for \(x\)'s in the interval \((-\infty, -1)\) (since that is where the \(y\)-values are negative).  
   k) The function is never decreasing.  
   l) Set function equal to 7 and solve for \(x\). \(x = 2\)

6. a) Domain \((-1, \infty)\)  
   b) Range: \((-\infty, \infty)\)  
   c) \(x \approx 2.17\)  
   d) \((0, -5)\)  
   e) \(x = -1\)  
   f) Graph
Mixed Review

1. Vertex \((2, -13)\)

2. \((x + 2)^2 + (y - 12)^2 = 100\)

3. Complete the square to get: \((x - 3)^2 + (y + 5)^2 = 26\)
   Center \((3, -5)\); radius = \(\sqrt{26}\)

4. Points are: \((2, 1), (-1, -2)\)

5. \(w = 250 + .04x\)

6. a) Set \(h(t) = 0\), and solve for \(t\). \(t = 7\) seconds.
   b) 196 feet (Maximum occurs at the vertex)

7. Solve the equation: \(x^2 + 8^2 = (x + 2)^2\). Area = 120 square miles

8. a) Graph using a graphing calculator
   b) As \(x \to \infty\), \(g(x) \to \infty\), and as \(x \to -\infty\), \(g(x) \to -\infty\).
   c) \((-2.11, 0), (.25, 0), (1.86, 0)\)
   d) \((0, 1)\)
   e) \((1.15, -2.08)\)