College Algebra

Review Problems for Final Exam

Equations

#1-10 Solve for the variable

1. \(2(x + 1)^2 - 4 = 0\)
2. \(\sqrt{2x + 5} = x + 3\)
3. \(3|9y + 4| = 21\)
4. \(x^4 - 6x^2 - 27 = 0\)
5. \(\frac{x^2}{x+15} = \frac{1}{2}\)
6. \(2k^2 + 8k = 7\)
7. \(\frac{x}{x-1} + \frac{1}{x+1} = \frac{7}{x^2-1}\)
8. \(3x^2 + 6x^2 = 9x + 18\)
9. \(1 + \log_2(x + 3) = 4\)
10. \(e^{2x+1} - 19 = 0\)

Inequalities

1. Solve the inequality: \(|2 - 5y| < 7\)
2. Solve the inequality: \(4x - x^3 \geq 0\)
3. Given: The expression \((2 - x)(x + 3)^2\). In what intervals is this expression less than or equal to zero?
4. What are the values of \(g\) for which the expression \(\frac{g-4}{1-g}\) is positive?
Function Topics

1. Name two ordered pairs which lie on the graph of \( y^2 = x + 1 \), and which show that this is not the equation of a function.

2. What is the domain of the function \( f(x) = \frac{1}{9 - x^2} \)?

3. What is the domain of the function \( F(x) = \sqrt{9 - x} \)?

4. What is the range of the function, \( y = 2x^2 + 5 \)?

5. Give an example for each of the following:
   a) A function whose domain and range are all real numbers
   b) A function which is increasing for all real numbers
   c) A function which is odd
   d) A function which is not one-to-one

6. Discuss the symmetry of each of the following relations (x-axis, y-axis, origin, or none of these):
   a) \( y^2 = 4 + x \)
   b) \( y = \frac{2}{x} \)
   c) \( y = x^4 - x^2 \)
   d) \( y = \sqrt{x} \)
   e) \( x^2 = 9 - y^2 \)

7a) Re-write the function, \( y = \sqrt{x} \) so that it is reflected in the x-axis.
   b) Re-write the function, \( y = \sqrt{x} \) so that it is reflected in the y-axis.
   c) Re-write the function, \( y = \sqrt{x} \) so that it is translated two units to the right.
   d) Re-write the function, \( y = \sqrt{x} \) so that it is translated three units up.

8. Re-write the function, \( y = x^3 \), so that it is vertically shifted two units down and horizontally shifted 4 units to the left.

9. Find the inverse function of \( f(x) = \frac{2x}{x+3} \).
10. If \( f(x) = x^2 \) and \( g(x) = \sqrt{x + 4} \), what is \( (g \circ f)(3) \)?

11. If \( G(x) = 17 - 3x \), find \( (G \circ G)(x) \).

12. Given the function \( Q(x) = x^2 - 3x \). Find the average rate of change of the function in the interval \([-1, 4]\).

13. Sketch the graph of the piecewise function \( P(x) = \begin{cases} 
  x - 2, & \text{if } x < 3 \\
  x^2, & \text{if } -3 \leq x \leq 2 \\
  5, & \text{if } x > 2 
\end{cases} \)

Label key points on the graph.
Exponents and Logarithms

1. Convert to exponential form: \( \ln x = 5 \)

2. Convert to logarithmic form: \( 10^{-3} = 0.001 \)

3. Solve for \( x \): \( 3(5^{x+1}) - 5 = 12 \)

4. Solve for \( x \): \( \log_4(1 - 8x) = -1 \)

5. Solve for \( x \): \( \log(x - 21) + \log x = 2 \)

6. Solve for \( y \): \( \log_5(2x) - \log_5(x - 3) = 1 \)

7. How long will it take for $4000 to triple if it is compounded continuously at 5% interest?

8. Given: \( f(x) = \log_4(x + 4) - 2 \)
   a) Find domain and range.
   b) Find the inverse function: \( f^{-1}(x) \)
   c) Find the domain and range of the inverse.
   d) For what value(s) of \( x \) is \( f^{-1}(x) > 0 \)

9. Assume we know that 8000 bacteria infected a patient’s body at 8 AM, and he is immediately treated at 8AM so that the bacteria begin to die at an exponential rate. We also know that at 11 AM there are 6000 bacteria left in the patient. Give answers to nearest thousandth.
   a) Find an exponential equation to model the number \( A \) of bacteria at time \( t \) hours, with 8AM being \( t = 0 \).
   b) Find the half-life of the bacteria.
   c) Calculate the number of bacteria left at 6 PM.
Analyzing Functions

1. Given: \( F(x) = (5x + 2)(x - 1)^3(x + 4)^2 \)
   
   a) What is the degree of this polynomial function?  
   b) What is the leading coefficient?  
   c) Describe the end behavior of the graph  
   d) List the zeros of \( F(x) \).  
   e) For each zero, state its multiplicity.  
   f) What are the coordinates of the y-intercept?  
   g) Sketch the graph.

2. Given: \( G(x) = -\sqrt{x + 3} + 4 \)
   
   a) Describe the vertical and horizontal translations.  
   b) Describe the type of reflection.  
   c) What is the domain of the function?  
   d) What is the range of the function?  
   e) Find the coordinates of the x-intercept.  
   f) Find the coordinates of the y-intercept.  
   g) Sketch the graph.

3. Given: \( g(x) = -2(x - 3)^2 + 4 \)
   
   a) What name is given to this type of function?  
   b) What is the domain?  
   c) What are the coordinates of the vertex?  
   d) What is the equation of the axis of symmetry?  
   e) Sketch the graph of the function by making a table of values.  
   f) What is the range?  
   g) Is this function one-to-one?  
   h) What are the coordinates of the x-intercepts?  
   i) What are the coordinates of the y-intercept?  
   j) Describe the end behavior.  
   k) For what values of \( x \) is the function increasing?  
   l) For what values of \( x \) is \( g(x) > 0? \)
4. Given: \( f(x) = \frac{5x - 1}{2x + 3} \)

a) What is the domain?
b) What name is given to this type of function?
c) What are the coordinates of x-intercept?
d) What are the coordinates of y-intercept?
e) Write the equations of horizontal and vertical asymptotes.
f) What is \( f(4) \)?
g) For what value of \( x \) is \( f(x) \) equal to 4?
h) Sketch the graph of the function by making a table of values.
i) Describe the end behavior.
j) What is the range?
k) Is this function one-to-one?

5. Given: \( h(x) = 2^{x+1} - 1 \)

a) What name is given to this type of function?
b) What is the domain?
c) What is the equation of the asymptote?
d) Find the coordinates of the x-intercept.
e) Find the coordinates of the y-intercept.
f) Sketch the graph of the function by making a table of values.
g) What is the range?
h) Describe the end behavior.
i) Is this function one-to-one?
j) For what values of \( x \) is \( h(x) < 0 \)?
k) For what values of \( x \) is the function decreasing?
l) For what value of \( x \) is \( h(x) = 7 \)?

6. Given: \( H(x) = 3 \log_2(x + 1) - 5 \)

a) What is the domain of the function?
b) What is the range of the function?
c) Find the zero of the function, to one decimal place.
d) Find the coordinates of the y-intercept.
e) What is the equation of the asymptote?
f) Sketch the graph.
Mixed Review

1. What are the coordinates of the vertex of \( f(x) = x^2 + 4x - 9 \)?

2. Write the equation of a circle whose radius is 10 and whose center is \((-2, 12)\).

3. Find the center and radius of the circle whose equation is:
   \[ x^2 + y^2 - 6x + 10y + 8 = 0 \]

4. Solve the system of equations:
   \[ y = x^2 - 3 \quad \text{and} \quad x - y = 1 \]

5. A sales job pays $250 per week plus 4% commission on all sales. Write a function for weekly salary, \( w \), in terms of weekly sales, \( x \).

6. An object is launched upward and its height above the ground is given by the function, \( h(t) = 112t - 16t^2 \).
   a) Find the time at which it strikes the ground.
   b) Find the maximum height.

7. Solve algebraically: A huge rectangular farmland is 8 miles wide and its diagonal is 2 miles more than its length. Set up an equation and solve it to find the area of the farmland.

8. Given the function: \( g(x) = x^3 - 4x + 1 \)
   a) Graph the function using an appropriate window.
   b) Describe the end behavior of the graph.
   c) Use ZERO to find all the zeros of the function.
   d) What are the coordinates of the y-intercept?
   e) Find the coordinates of the minimum point.