Newton’s Method Using the Graphing Calculator

(This technique is used for approximating the zeroes of a function)

Initial Set-up

- Press Y =
- Set \( Y_1 \) = original function
- Set \( Y_2 \) = the function’s first derivative
- QUIT → Press 2nd MODE

First Iteration

In the next command, replace your initial value wherever you see “#”

- \( # - \frac{Y_1(#)}{Y_2(#)} \)  Note: \( Y_1 \) is obtained by entering the following:
  VARS,  →,  ENTER,  1
- ENTER  (This will produce your 1st iteration on the screen)

Subsequent Iterations

- \( \text{ANS} - \frac{Y_1(\text{ANS})}{Y_2(\text{ANS})} \)  Note: \( \text{ANS} \) is obtained by entering:
  2nd (-)
- ENTER  (This will produce your 2nd iteration on the screen)
- ENTER  (This will produce your 3rd iteration on the screen)

Continue to press the ENTER key as often as needed. Stop when the digits duplicated from one answer to the next are to the desired accuracy.
Example: Find the value of $\sqrt[6]{2}$ to nine decimal places.

This is equivalent to finding the zeroes of the function $f(x) = x^6 - 2$
(Hint: Set the left side of the equation to 0 and solve for $x$ to see why)

- Press $Y =$
- Set $Y_1 = x^6 - 2$
- Set $Y_2 = 6x^5$
- QUIT $\Rightarrow$ Press 2nd MODE

Note: This example arbitrarily chooses an initial value of 1.

- $1 - Y_1(1)/Y_2(1)$ Note: $Y_1$ is obtained by entering the following:
  VARS, $\Rightarrow$, ENTER, 1
  ENTER
  The initial estimate is: $1.1\overline{6}$

- $ANS - Y_1(ANS)/Y_2(ANS)$ Note: $ANS$ is obtained by entering:
  2nd (-)
  ENTER
  The 2nd iteration = 1.126443678
  ENTER
  The 3rd iteration = 1.122497067
  ENTER
  The 4th iteration = 1.122462051
  ENTER
  The 5th iteration = 1.122462048
  ENTER
  The 6th iteration = 1.122462048

Solution: The value of $\sqrt[6]{2}$ to nine decimal places = $1.122462048$