Review of Plane Geometry

Points, Lines, and Rays

A point has no length or width. • A line is straight with unlimited length in two directions but no width.

A ray has as starting point and unlimited length in one direction but no width.

Angles, Triangles

An angle is formed by two rays with the same beginning point called the vertex.

Angles are measured in degrees or radians. Two lines are perpendicular if their intersection forms a right angle (90°).

Complementary angles add to 90°.

\[ \angle a + \angle b = 90° \]

Supplementary angles add to 180°.

\[ \angle a + \angle b = 180° \]

A straight angle measures 180°.

Parallel lines do not intersect.

The opposite angles formed by a transverse line intersecting two parallel lines are called vertical angles. 1 and 4 are vertical angles. So are 2 and 3, 5 and 8, and 6 and 7.

Vertical angles are congruent (have the same measure). So angle 1 = angle 4, angle 2 = angle 3, angle 6 = angle 7, and angle 5 = angle 8.

Angles 3, 4, 5, and 6 are called interior angles because they are between the parallel lines. Angles 1, 2, 7, and 8 are called exterior angles because they are outside the parallel lines.

Alternate interior angles are congruent (angle 3 = angle 6 and angle 4 = angle 5).

Alternate exterior angles are congruent (angle 1 = angle 8 and angle 2 = angle 7).

Interior angles on the same side of the transversal are supplementary. So angle 3 + angle 5 = 180° = angle 4 + angle 6.

A triangle has three sides and three angles. The sum of the angles of a triangle is 180°.

In an equilateral triangle all three sides are the same length and all three angles are also congruent.

An isosceles triangle has two congruent angles and sides.

Right triangle: One of the angles is 90°. This is indicated by a “square” in the angle.

Pythagorean Theorem: \( a^2 + b^2 = c^2 \)
Area of a triangle is \( \frac{1}{2} \) base x height.

In similar triangles, corresponding angles are congruent. Thus, similar triangles have the same shape, and their corresponding sides are proportional.

\[
\frac{A}{a} = \frac{B}{b} = \frac{C}{c}
\]

**Quadrilaterals**

A quadrilateral has four sides (and four angles). The four angles of a quadrilateral add up to 360°.

A trapezoid has two parallel sides.  

\[
\text{Area} = \frac{1}{2}(b_1+b_2)h
\]

A parallelogram has two sets of parallel sides.

\[
\text{Area} = bh
\]

A rectangle is a parallelogram with each angle 90°.

A square is a rectangle with each side the same length.

\[
\text{Area} = lw
\]

**Circles**

A circle is the set of all points that are the same distance from a point called the center. The distance from the center to any point on the circle is the radius of the circle. The diameter is a line segment that goes through the center with endpoints on the circle. So a diameter = two radii.

The circumference is the length of the circle if it is cut once and formed into a straight line. A triangle inscribed with one side a diameter is a right triangle.

\[
\text{Circumference} = 2\pi r = \pi d
\]

\[
\text{Area} = \pi r^2
\]
Angle Measure

A circle has 360°. If a circle has \( radius = 1 \) and an angle is formed with the vertex of the angle at the center of the circle, the **radian measure** of the angle is the number of \( radius \) lengths in the **arc** the angle subtends (or cuts out) on the circle.

There are \( 2\pi \) radius lengths in any circle (the circumference = \( 2\pi \times \) radius), so for a circle of radius \( = 1 \), the circumference is \( 2\pi \) radians. Radians are abbreviated as \( rad \).

\[
\begin{align*}
90° &= \frac{\pi}{2} \text{ rad} \\
180° &= \pi \text{ rad} \\
270° &= \frac{3\pi}{2} \text{ rad} \\
360° &= 2\pi \text{ rad}
\end{align*}
\]

To convert radians to degrees, multiply by \( \frac{180°}{\pi} \)

Example: \( 1 \text{ rad} = \frac{180°}{\pi} \approx 57.3° \)

To convert degrees to radians, multiply by \( \frac{\pi}{180°} \)

Example: \( 90° = 90° \cdot \frac{\pi}{180°} \text{ rad} = \frac{\pi}{2} \text{ rad} \)