Geometric Optics

Purpose

In this lab we will measure the index of refraction of a translucent material by following the paths of light rays through a rectangular prism made of the material. In so doing we will confirm Snell’s Law for the index of refraction of light. We will also measure the index of refraction for the material using two other effects: total internal reflection and the minimum angle of deviation of a triangular prism.

Finally, we will trace light ray paths through concave and convex lenses and from mirrors to observe how these objects affect the directions of the rays.

Principles

Although light travels as a wave, for many purposes its wave nature is not evident because its wavelength is so short (on the order of $10^{-7}$ m for visible light). When studying the behavior of light as it passes through lenses or reflects off mirrors, we can think of light as beams traveling in straight lines, called rays. This is the approach taken in geometric optics. The rays are simply the direction in which the waves are traveling.

There are two effects that concern us in geometric optics: reflection and refraction. Both are cases where the light rays change direction because the light has encountered a new medium.
Reflection

We are all familiar with the fact that light can reflect off of certain surfaces – metals and glass, for instance. In this case, the light rays simply bounce off the surface. You also may be familiar with the fact that the reflected ray will make the same angle with the surface as the incident ray did, but transposed across the normal to the surface (see the top part of Diagram 1). We say, “the angle of incidence is equal to the angle of reflection.”

\[ \theta_i = \theta_L \]

\[ n_i \sin \theta_i = n_R \sin \theta_R \]

You should keep in mind that light reflects off any surface to some degree, especially translucent materials like glass or acrylic. A translucent material allows most of the light to pass through, but some of it will be reflected, as you can confirm by looking at the surface of any window. Also, each surface of the material will reflect some of the light. For example, light reflects both from the front surface of the window – before it has entered the glass – and from the back surface – after it has traveled through the glass.

A mirror works as it does because of an opaque backing that prevents the transmission of the light. In this case most of the light is reflected (some is absorbed by the material).
Refraction

When light passes from one translucent medium to another, for instance from air to water or from air to glass, its rays will change direction at the interface between the two media. This change in direction of the light rays is called refraction (see the bottom part of Diagram 1.). We are all familiar with this effect: a pencil set upright in a glass of water will appear bent because rays traveling from the underwater part of the pencil change direction when they pass from the water to glass and then from the glass to the air.

Refraction is a consequence of the fact that the speed of light changes when light enters a new medium. Light travels through vacuum at $3.00 \times 10^8$ m/s, but it slows down to about $2 \times 10^8$ m/s in glass. Since the wave fronts move slower through glass than they do through air, the directions of the wave fronts – the rays – necessarily turn (see Diagram 2).

We define the index of refraction of a medium as the ratio of light’s speed in vacuum, $c$, to light’s speed in the medium, $v$:
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\[ n = \frac{c}{v} \]

Note that the index of refraction of a vacuum is 1. The index of refraction of air is 1.0001, since light slows down only slightly when it travels in air. In this lab, we will approximate the index of refraction of air as 1.00.

It follows from the above definition and from the geometry of the refraction (illustrated in Diagram 2) that

\[ n_i \sin \theta_i = n_r \sin \theta_r \]  

where the subscripts \( I \) and \( R \) refer to the incident and refracted rays, respectively, and the angles are those the rays make with the normals to the interface between the two media. This relationship is known as Snell’s Law.

We can use this to determine the index of refraction of a material. In this lab we will direct light at various angles through a rectangular prism made of acrylic or glass. By measuring the incident and refracted angles of the light beams and using the fact that the index of air is approximately 1.00, we can calculate the index for the material.

Note that Snell’s Law works in either direction. For instance, in Diagram 2, we could reverse the direction of the rays if we also interchange the labels \( \theta_i \) and \( \theta_r \). Also note that if the incident ray is perpendicular to the surface (\( \theta_i = 0 \) degrees), then the “refracted” ray is also perpendicular to the surface (\( \theta_r = 0 \) degrees) and the ray is not bent.

**Total Internal Reflection**

When light passes through a surface, some fraction of the incident beam will reflect from that surface (refer again to Diagram 1). The intensity of the reflected beam increases as the angle of incidence increases. If light is passing from a dense medium to a less dense one (higher refractive index to lower), at a certain critical angle of incidence, all of the light is reflected and none is transmitted.

This situation is known as total internal reflection. This occurs when the refracted beam is parallel to the surface (\( \theta_r = 90 \) degrees). At higher angles of incidence, it is not possible for the beam to pass through the surface and all of the light is reflected from it.

This gives another way to measure the index of refraction of a material. Refer to Diagram 3, which illustrates a beam of light passing through a semi-circular lens, or D-lens. Here, we consider the beam as it exits the lens rather than enters it so that the incident region is inside the material and the refracted region is outside.
The incident ray will not be refracted when it enters the D-lens if it strikes the surface normally (we will arrange for this to be so). It will be partly refracted and partly reflected when it tries to exit the plane surface, so that two beams will emerge. If we swivel the lens relative to the incoming beam so that $\theta_I$ increases, at the critical angle the refracted ray will disappear and the reflected ray will noticeably increase in intensity.

At the critical angle of incidence, the angle of refraction, $\theta_R$, is 90 degrees. Since $\sin(\theta_R) = 1$ and since the index of refraction of air is 1, we have from Snell’s Law:

$$n_I = \frac{1}{\sin(\theta)}$$

Here, $n_I$ is the index of refraction of the D-lens.

**Minimum Angle of Deviation**

When a light ray passes through a triangular prism, it is refracted twice: once when it enters the prism and again when it leaves the prism (see Diagram 4). The angle of deviation of the incident ray is the change in its direction after passing completely through the prism. The magnitude of the deviation depends on the apex angle of the prism (the angle between the two sides involved), the angle of incidence of the original beam and the index of refraction of the prism material.

At a certain angle of incidence, the deviation will be a minimum. It can be shown that the index of refraction of the prism material is related to the minimum angle of deviation by
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\[ n = \frac{\sin\left(\frac{\phi + \theta_{\text{min}}}{2}\right)}{\sin\left(\frac{\phi}{2}\right)} \]

(3)

where \( \phi \) is the apex angle of the prism.

Diagram 4: Angle of deviation

This gives us a third way of measuring the index of refraction.
Part 1: Snell’s Law

We will use Snell’s Law to measure the index of refraction of acrylic, a type of plastic. We will shine laser light through a rectangular block, a triangular prism and a semi-circular lens made of this material. Each will give us a separate measurement of the material’s index of refraction.

A laser is used since this gives a well-focused beam of light with a single wavelength. Ordinary light tends to disperse (spread out) rapidly and is composed of a mixture of wavelengths. Different wavelengths refract at different angles, so measurement made with ordinary light would not give a clearly defined value. At the same time, this means we will be determining the index of refraction of the material at the particular wavelength of the laser light.

The experiments can be outlined as follows:

1. **Rectangular prism.** Direct the laser beam at the front surface of the block and measure the angles the incident and refracted rays make with the surface normal. The index of refraction can be calculated from Snell’s Law. We will do this for six angles of incidence.
2. **D-lens.** Direct the laser beam at the semi-circular surface and determine the critical angle at which total internal reflection occurs. The index of refraction can be calculated from this angle.
3. **Triangular prism.** Direct the laser beam through the prism and determine the minimum angle of deviation. The index of refraction can be calculated from this angle.

**Equipment**

- Rectangular and triangular prisms
- D-lens
- Laser
- Corkboard
- T-pins and/or metallic probe
- Hand-held screen or index card
- Drawing or bond paper
- Protractor
- Ruler
- Table lamp
- Thumb tacks or push pins.

**Procedures**

**Note:** Never look into the laser nor directly into a reflection from it. Be careful not to point the laser at or near anyone’s face. The beam can damage your eye.
Part 1: Snell’s Law

Handle all the lens and prisms in this lab with care. Hold them by their tops and bottoms, or by edges not used in the experiments. Do not touch the front or back surfaces.

Dim the room lights so that the path of the laser inside the prisms is visible. Use a table lamp for written work.

1. **Rectangular Prism**

   a. *Set up an angle scale on paper (see diagram below):*

      - Draw a horizontal line across the middle of a piece of white paper. We will place the face of the rectangular prism along this line.
      - Mark a point in the middle of the line. This will be the target point for the laser beam.
      - Draw a semi-circle around this point with a protractor, using the horizontal line as the base. We will measure the incident angles along this semicircle.
      - Mark the 90-degree point on the semicircle. This will mark the normal to the surface.
      - Draw another semi-circle on the other side of the line and mark its 90-degree point. We will measure refraction angles along this semi-circle.
      - Draw a straight line through the two 90-degree points and through the target point. This is the normal to the lens surface.
      - Tack the paper to the corkboard and place the rectangular prism on the paper with one long face along the horizontal line, centered at the target point. Outline the prism on the paper so that you can reposition it when needed.

   b. *Center the laser beam along the surface normal.*
Part 1: Snell’s Law

- Turn on the laser and direct its beam along the 90-degree line so that it hits the target point.
- Use a T-pin to locate the beam (it is not visible in air).
- Use the reflection from the front face of the prism to square the beam. The reflection will travel back into the laser aperture when the beam hits the prism face dead-on.
- Once you have the beam perpendicular to the front face of the prism, do not move the laser. Instead, swivel the corkboard.

c. Find the path of 6 different rays through the prism.

- For each new ray, swivel the corkboard by about 8 degrees so that the beam hits the target point at a new angle. Reposition the corkboard as necessary so that the beam hits the target point. *Always use the same target point* and do not disturb the alignment of the prism with the paper. You can tell where the beam is entering and exiting the block by the small reflection spots on the front and rear faces. Keep the front spot directly above the target point. You should look vertically downward to align the spot (i.e., avoid parallax).
- Use the T-pin to locate the incoming beam along the front semi-circle. Push the pin through the paper to mark the spot.
- Use the T-pin to mark the spot where the beam hits the back edge of the prism.
- Put the prism back in its case when finished.

Analysis:

1. Draw the incident and refracted rays with a ruler. Extend the refracted rays to the back semi-circle.
2. Measure the angles of incidence and refraction for each ray to the nearest tenth of a degree. The angle of refraction is the angle the ray makes with the surface normal as it leaves the target point (i.e., refraction from the front face of the prism). Record this data.
3. Calculate the sines of the angles of incidence and refraction and record these to three significant figures.
4. Use Snell’s Law to calculate the index of refraction of the block for each ray. Record your results.
5. Average the resulting indices. Take the percent error of your result with the manufacturer’s stated value for the index of refraction for this material: 1.50.

2. Total Internal Reflection
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**Part 1: Snell’s Law**

*a. Set up a diagram on paper.*

- Again, draw a horizontal line across a piece of white paper. Draw a line perpendicular to the horizontal line at its center. This will represent the normal to the straight face of the D-lens. The intersection of the two lines will be the target point.
- Tack the paper to the corkboard, center the lens along the horizontal line and outline it.

*b. Center the beam.*

- Direct the laser so that it travels along the normal line and exits the lens at the target point. Again, use T-pins to locate the beam, and use the reflections of the beam to square it up.

c. *Rotate the corkboard (with the lens on it) and find the refracted and reflected rays.*

- Use an index card to track two exiting beams: a refracted beam coming out of the back face of the lens and a reflected beam coming out of the front face. Make sure the beam keeps hitting the curved surface normally as you rotate the lens.
- Note (in your lab notebook) what happens to the intensity of the reflected and refracted beams as the lens is rotated.

d. *Find the critical angle.*

- When the refracted ray first disappears, use a T-pin to mark the positions of the incoming ray from the laser and the outgoing reflected ray.

**Analysis:**

1. Draw the ray paths of the incident and reflected rays at the critical angle.
2. Measure the angle of incidence and the angle of reflection at the critical angle.
3. Calculate the index of refraction of the lens material using Snell’s Law. Note the beam is being refracted at the back face here, so the incident beam is coming from within the lens and the refracted beam is exiting into the air. This means that \( n_I \) and \( n_R \) are reversed from what they were in the first experiment.
4. Calculate the percent error from the given index of refraction for the D-lens.
Part 1: Snell’s Law

5. Describe the relationship between the angle of incidence and the angle of reflection and calculate the percent difference between the measured values for these angles.
6. Describe what happens to the intensities of the two beams as the critical angle is reached.

3. Minimum angle of Deviation of a Prism

a. Draw a line through through the middle of the long side of a sheet of paper to represent the path of the laser beam. Tack the paper to the corkboard and direct the laser beam along the line.

b. Place the triangular prism on the paper so that the beam passes through (refracts through) two of the prism’s faces. (Reflections from internal faces are not what we are looking for.) The exact orientation does not matter. Use a screen or index card to determine the direction of the exiting beam.

   Slowly rotate the prism, keeping the point of entry of the beam constant. Track the exiting beam with the screen. Find the orientation of the prism at which the exiting beam makes the smallest angle with the incident beam. This is the minimum angle of deviation.

c. Trace the outline of the prism at the minimum angle and mark the direction of the exiting beam by pushing a T-pin into the paper at the point of exit and at a second point beyond the prism.

Analysis.

1. Indicate the direction of the incident beam with an arrow on the horizontal line.
2. Draw the path of the beam within the prism by connecting the entry and exit points.
3. Draw the path of the exiting beam. Use dashes to extend this line to intersect with the incident ray.
4. Measure the minimum angle of deviation and the apex angle of the prism. Record these values.
5. Calculate the index of refraction using equation (3), and the minimum angle of deviation.
6. Compare your result to the stated value of the prism material (1.50).
Part 2: Ray Paths through Lenses and Mirrors

In this part of the lab we will observe and record ray paths through “cylindrical” lenses and mirrors. Because of their shape, these objects will make light beams either converge to a focal point, or diverge from a focal point. These properties can be used to form images of distant objects. We will investigate image formation in the next lab. Some terminology:

- A **convex lens** is a converging lens. Parallel rays entering the lens will converge to a focus on the other side of the lens.
- A **concave mirror** is a converging mirror. Parallel rays striking the surface will converge to a focus in front of the mirror.
- A **concave** lens is a diverging lens. Parallel rays entering the lens will appear to diverge from a point in front of the lens.
- A **convex mirror** is a diverging mirror. Parallel rays striking the surface will appear to diverge from a point on the other side of the mirror.
- The **focal length** of a lens or mirror is the distance from the focal point to the center of the lens or mirror, along the axis of symmetry.

**Procedures**

Plug in the light source and lay it flat on the table. Set the sliding aperture so that it emits 5 beams. Again, darken the room lighting. For each of the lenses and mirrors supplied:

1. Place the lens or mirror on a sheet of paper and trace its outline.
2. Set the light source on the paper about 5-10 cm in front of the lens or mirror. Direct the beams parallel to the object’s axis of symmetry, so that the center beam is along the lens or mirror axis, and the two other beams are on either side.
3. With a pencil, mark the locations of the incoming and exiting beams. You will need to make four marks for each beam: two to indicate the incident direction and two to indicate the refracted or reflected direction. Mark the centers of the beams. Assign a number to each ray and label each mark with the ray number so you can keep track of which ray is which.

**Analysis**

1. Draw the ray paths from the source and through the focal points. For the diverging lens and mirror, use dashed lines to extend the paths back to the focal points.
2. Measure and record the focal lengths of each lens or mirror. Measure from the center of the lens or mirror to the point where the rays intersect. If the rays do not meet at exactly the same point, determine an average focal point.
3. Determine the radius of curvature of each of the objects and find a relationship between the focal length and the radius of curvature.
Data & Analysis