Capacitors

A capacitor is an electrical component that stores charge. The simplest capacitor is just two charged metal plates separated by a non-conducting material:

In the diagram above, each metal plate has the same magnitude of charge, \( Q \), but opposite charge polarity. The non-conducting material is the air between the plates. Because of the charge separation, an electric field is set up, and there is a potential difference between the plates: \( V_b - V_a \).

We charge a capacitor by connecting the two plates to a potential difference, such as a battery:
The battery pumps charge from one plate to another. The plates end up with the same potential difference between them as the battery.

We define the *capacitance* $C$ of a capacitor as the ratio of the magnitude of the charge $Q$ on each plate to the voltage difference between them.

$$C = \frac{Q}{V}$$

This ratio depends on the shape and size of the capacitor and the separation distance between its plates. The unit of capacitance is the *farad*, which is equivalent to *charge per volts*.

In practical capacitors, the plates are often rolled up into a cylinder, with plastic used as the non-conducting material (called the “dielectric”) between them.

**Discharging capacitor**

If we connect a resistance across a charged capacitor, the capacitor then acts as a power source for the resistor. Current will flow through the resistor until the charge on the capacitor’s plates have neutralized:
However, the discharge will not occur all at once. It takes time for the capacitor to discharge. Also, the current will not be steady: it will decrease over time as the charge on and voltage across the capacitor decrease. This is an example of a \textit{time-dependent} current.

Kirchhoff’s Loop Rule still applies: The voltage across the capacitor (given by $V_C = \frac{Q}{C}$) must equal the voltage across the resistor ($V_R = IR$) at any time. To find the charge $Q$ as a \textit{function} of time, we note that the current relates to the capacitor’s charge by: $I = -\frac{\Delta Q}{\Delta t}$. (The minus sign arises because the charge decreases as the current increases.) So:

$$\frac{Q}{C} = IR = -\frac{\Delta Q}{\Delta t}R$$

Or

$$\frac{Q}{RC} = -\frac{\Delta Q}{\Delta t}$$

It is shown in Calculus that this means the charge on the capacitor is a decreasing exponential function of time:

$$Q = Q_0e^{-\frac{t}{RC}}$$

The voltage across the capacitor is just $Q/C$:

$$V(t) = V_0e^{-\frac{t}{RC}}$$
A decreasing exponential function looks like this:

The factor $RC$ in the exponential $e^{-t/RC}$ is called the characteristic time, or the time constant for the circuit. You can show that the product $RC$ has units of time.

When the capacitor has discharged for one characteristic time, then $t = RC$ and the exponential will have the value: $e^{-1} = 0.368$. The voltage will have decreased to 36.8% of its original value. In the next interval of $RC$ seconds ($t = 2RC$), the capacitor will decrease another 36.8%, and so on.

We can determine the time constant, and thus the capacitance in a circuit, by plotting the voltage across the capacitor as a function of time. The voltage will be $0.368V_0$ at the time $RC$ seconds. For instance, in the diagram above, this appears to occur at about 10 seconds. If the resistance is, say 20,000 ohms, then the capacitance must be $10 \text{ sec}/20,000 \text{ ohms} = 5 \times 10^{-4}$ farads, or 500 microfarads.

This predictable time-dependence means capacitors can be used as timers, as well as charge or voltage sources. For instance, flashing lights are often powered by discharging capacitors.

In Part 1 of the lab, we will time the discharge of a capacitor using two different resistances. Plotting the capacitor voltage over time, we can determine the time constant of the circuit and thus the value of the capacitor.
Capacitors and Alternating Voltage

In the second part of the lab, we will look at the charge-discharge cycle of two fast capacitors (small capacitance) connected to a square wave voltage. Since the time constants will be in the millisecond range, we will use an oscilloscope to time the voltage decay of the capacitors.

A square wave voltage reverses its polarity at regular intervals:

![Square Wave Diagram]

The current in the circuit will thus alternate in direction at a frequency that can be set, and the capacitors will alternately charge and discharge. The exponential character of each part of the cycle will be visible on the oscilloscope screen. It will be easiest to measure the “half-lives” of the capacitors – the time it takes them to decay to half their initial voltage values. It is easy to show that the half-life, $t_{1/2}$, is related to the time constant, $RC$, by

$$ t_{1/2} = RC \ln 2 $$
Procedures

Part 1. You will perform the following for two different resistors:

1. Measure and record the resistance of the resistor. Also record the listed value of the capacitance on the capacitor.

2. Set up the circuit in Diagram 1. Note: If your capacitor has an arrow on it, connect it so that the arrow points towards the ground end of the circuit.

![Diagram 1](image)

The switch can simply be an unplugged banana wire. To charge the capacitor, complete the connection to the power supply. To allow the capacitor to discharge through the resistor, simply disconnect the resistor-capacitor from the power supply. The voltmeter is connected across the capacitor.

**Note:** The voltmeter is a resistor also, so in the above circuit we actually have two resistors in parallel connected across the capacitor. However, the resistance of the voltmeter is so large that the equivalent resistance of the discharge circuit is essentially the resistance of the resistor.

3. Set the output of the power supply to 10.0 volts. Charge up the capacitor by connecting to the power supply. It will charge to 10.0 volts.

4. When you are ready, unplug the resistor-capacitor loop from the power supply and start the timer.

5. Record the voltage across the capacitor at 15 second intervals until the voltage has decreased to less than 0.1 volts, or for 9 minutes, whichever comes first. (Nine minutes fits conveniently on our graph paper.)

Analysis

1. Plot the capacitor voltage versus time for each resistor. Make your graph one full page in size. You can run the plots for both resistors on the same sheet.
2. Annotate on your graph the voltage value at .368V0. For each resistor, annotate the time constant value.
3. From each plot and its time constant value, calculate the capacitance of the capacitor.
4. Average the two values and take a percent difference with the listed value on the capacitor.
5. Show that the product RC has units of time.

Part 2: You will perform the following for two different capacitors

1. First, connect the signal generator to channel 1 of the scope. Turn both on and set the SG’s output to about half-amplitude and square wave output. Start with a frequency of about 500 hz. Adjust the scope until you get a good display. The square wave voltage will look like two dashed lines.
2. Once the SG’s output is adjusted, connect the circuit below: the resistor and capacitor in series with the SG as power source.

   ![Resistor SG Capacitor](image)

3. Connect channel 1 of the scope across the capacitor. Adjust the volts/div and time/div knobs to get a good trace.
4. Now find a frequency range where the capacitor fully charges and discharges in each cycle. This will be at low frequencies. (At high frequency, the capacitor does not have time to charge fully.)
5. Adjust the frequency and the display so that one charge-discharge cycle fills the screen. You will have to coordinate the frequency output of the SG with the time/div knob on the scope.
6. For each capacitor, measure the time it takes to decay to ½ its maximum voltage.

Analysis

1. Calculate the time constant for each capacitor from the ½ decay time.
2. Calculate the capacitance from the time constant and the resistance.
3. Take the percent difference between your calculated values and the values listed on the capacitors.
4. Show that $t_{1/2} = RC \ln(2)$