In this laboratory we will observe and measure standing waves on a stretched string. We will find the string’s resonant frequencies and use this information to determine its mass density.

**Waves on Strings**

We will use a mechanical vibrator to vibrate a stretched string. Using a signal generator, the frequency of vibration of the vibrator and of the string can be controlled. When the frequency is just right, a standing wave pattern can be set up on the string. These patterns look something like the following:

![Standing wave patterns](image)

The loops represent the envelopes of vibration of the string. The end-points of the loops – where the string is not vibrating – are called nodes. The index \( n \) refers to the number of nodes in the pattern, not counting the first, and denotes the mode of vibration of the string. The set of patterns denoted by all integer \( n \) (\( n = 1, 2, 3, \ldots \text{ infinity} \)) are called the natural modes of vibration of the string.

The frequency of the vibrator is called the input or driving frequency. At an arbitrary driving frequency, the string will vibrate in a complex combination of its natural modes and no pattern will be visible. At certain frequencies, however, a single natural mode will be become dominant and the amplitude of the string’s vibration will be large. This
condition is called resonance and the string frequency of vibration is called a resonant frequency.

**Standing Waves**

It can be shown with Newton’s Laws that a vertical displacement of a horizontally stretched string will propagate down the string with speed

\[ v = \sqrt{\frac{T}{d}} \]

where \( T \) is the tension and \( d \) the mass per unit length of the string. Once displaced and released, the string will continue to vibrate because of its elasticity, sending new displacements down the string. These propagating displacements constitute a wave traveling along the string with speed \( v \).

For any wave, the speed \( v = \lambda f \), where \( \lambda \) is the wavelength (the length of one cycle) and \( f \) is the frequency of vibration (the number of cycles per unit time). These traveling waves reflect off the boundaries of the string and reverse direction, superimposing on new waves coming down. A complex pattern of superimposed traveling and reflected waves results.

In general, the superposition tends to damp out the vibration of the string. When the wavelength is just right, however, the traveling waves match up with the reflected waves and they reinforce each other. The result is one of the standing wave patterns illustrated above.

The condition for a standing wave is that an integer number of half-wavelengths will fit into the length \( L \) of the string: \( L = n \frac{\lambda}{2} \), where \( n \) is the number of half-wavelengths.

Since \( \lambda = \frac{v}{f} \), this means that

\[ L = n \frac{v}{2f} = \frac{n}{2f} \sqrt{\frac{T}{d}} \]

or

\[ f_n = \frac{n}{2L} \sqrt{\frac{T}{d}} \quad \text{(for } n = 1, 2, 3, \ldots \text{)} \]

The resonant frequencies thus depend on the tension in the string, its mass density, and its length.


Set up the String and Mechanical Vibrator

1. Set up the pulley at the end of a lab table using a table clamp.
2. Attach a pendulum clamp to a table stand and set it on the table about a meter away from the pulley. The pendulum clamp should point towards the pulley.
3. Get a piece of string about 1.2 m long. Clamp one end to the pendulum clamp. Tie a loop in the other end and strand this over the pulley.
4. Put 200 grams on a mass hanger and hang this from the loop in the string.
5. On top of the mechanical vibrator is a lever with positions marked “locked” & “unlocked”. LOCK the driver while you are setting up, but remember to UNLOCK before you turn everything on.
6. Place the vibrator about 7 cm from the pendulum clamp end of the string. Let the string run through the U-notch in the wave driver post.
7. Arrange the height of the string so that it is level along its length and runs along the bottom of the U-notch to the pendulum clamp.
8. Connect the signal generator to the vibrator with the red and black banana wires. Connect to the signal generator’s “LO Ω” and “Gnd” output sockets. The vibrator has banana plug sockets on its side.

Adjust the Height of the String

1. UNLOCK the vibrator.
2. Plug in the signal generator and turn it on. Set its amplitude knob to mid-range. Turn down its frequency to about 1 hertz.
When you turn on the signal generator you will hear a sound coming from the vibrator. The vibrator is just a stereo speaker with a post attached to its diaphragm. When the diaphragm vibrates a sound is produced at that frequency. The vibration of the diaphragm drives the vibrator’s post, which in turn vibrates the string.

At 1 hertz, you can see what is going on, and you can now better adjust the height of the string. You want the post to push up the string as it ascends, but you also want it to release the string on the way down, so that the string can vibrate freely. Arrange the string height so that the vibrating post is out of contact with the string when it is about ¾ down:

At higher frequencies, the amplitude of vibration of the post will decrease and the post may cease to give the string a good push. If this occurs, lower the pendulum clamp slightly to reposition where the string runs through the notch.

Find Resonant Frequencies

1. Put 200 grams on the hanger. Measure the full length of the string and record the length and the total hanging mass.
2. Use the “Adjust” or “Frequency” knob on the signal generator to increase the frequency of vibration. Gradually raise the frequency until you find the 1st resonance. Carefully adjust the frequency at resonance so that you get a pattern that is (1) large amplitude, (2) stable. Record the frequency.

   We will not use the 1st resonance in calculating because it is uncertain what the true vibrating length of the string is. (The vibrator interferes with the free vibration of the string.) However, you can use this frequency as a guide in finding the higher resonances.

3. Find resonance modes 2, 3, 4, 5. To hone in on these resonances, it is useful to watch one of the nodes. At resonance, the amplitudes of the anti-nodes will be as large as possible and the nodes will be clearly defined.
4. For resonances 2, 3, 4, 5, measure the length of one segment. This length is one half-wavelength ($\lambda/2$).
5. Find the highest resonance that you can see visually. Record its mode and frequency.
6. Put 500 grams on the hanger (change the string tension). Find resonances up to the fifth mode and record the frequency and the length of one segment.
7. Shorten the string length to 60 cm, put 200 grams on the hanger, and find resonances up to the fifth mode, as above.
8. Weigh a 3-meter length of string.

Analysis

1. Calculate the mass per unit length of the string ($d$).
2. For each of the resonances, calculate the expected frequency, using the resonant frequency formula. Use your measured value for the string mass density and your measured value for the full length of the string. The tension $T$ in the string is the weight (in newtons) of the hanging mass and the mass hanger.
3. For each of the 2, 3, 4, 5 resonances, calculate the mass density of the string from the resonant frequency formula. Use your measured length for a vibrating segment for this. The vibrating length of the string, $L$, is the number of segments ($n$) times the segment length.
4. Average your results and compare to your measured mass density.
DATA

Measured string mass density: _______________________

Length of String: _____________

Tension: _____________

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Average Mass Density

% error