The Ballistic Pendulum

Experimental Objectives
The objective of this experiment is to study the law of conservation of momentum. We will apply the principle of conservation of linear momentum to a case of a perfectly inelastic collision in order to determine the initial velocity of projectile using a ballistic pendulum.

Overview
The Principle of Conservation of Linear Momentum states that if no external forces acting on a system of two or more objects, the total momentum of the system remains constant. The linear momentum \( p \) of an object of mass \( m \) and moving at linear velocity \( v \) is defined as the product of its mass and velocity \( (p = mv) \). From the conservation of linear momentum, the total momentum of the system before collision is equal to the total momentum of the system after collision. In a perfectly inelastic collision of two objects, the two colliding objects stick together after the collision and move off as one body, with the same final velocity.

The ballistic pendulum device used in this experiment consists of a combination of a ballistic pendulum arm with a massive cylindrical bob and a spring gun. A projectile, which is usually a metal ball, is fired by the spring gun into the pendulum bob and is held there by a spring catch. After the collision, the pendulum and the ball move together with the same velocity. Therefore, according to the conservation of the linear momentum of a system,

\[
m_b v_b = (m_b + M_p) V_f \quad \text{----- (1)}
\]

where \( m_b \) is the mass of the ball in kilograms, \( v_b \) is the initial velocity of the ball in m/s just before the collision. \( M_p \) is the mass of the pendulum and \( V_f \) is the final velocity of the combined pendulum and the ball after the collision.

After the collision, the pendulum with the embedded ball, swings about its point of support. The center of gravity of the pendulum combination rises through a vertical height \( h \) as it swings. During the upward swing, the kinetic energy of the system decreases while the potential energy increases. If we assume negligible loss of mechanical energy due to friction at the point of support, then by the law of conservation of energy, the kinetic energy at the bottom of the swing is equal to the potential energy at the highest point of the swing. The energy equation is given as:

\[
\frac{1}{2} (m_b + M_p) V_f^2 = (m_b + M_p) gh \quad \text{----- (2)}
\]

where \( g \) is the known acceleration due to gravity and \( h \) is the maximum height of the pendulum above its equilibrium position.

By combining equations (1) and (2) above, the initial velocity of the ball before the collision can be calculated.
**Apparatus**

<table>
<thead>
<tr>
<th>Photogate</th>
<th>Calipers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance</td>
<td>Three-legged base ring stand</td>
</tr>
<tr>
<td>Metric ruler</td>
<td>Universal Table Clamp</td>
</tr>
<tr>
<td>Various clamps</td>
<td>Blackwood Ballistic Pendulum</td>
</tr>
</tbody>
</table>

**PROCEDURE:**

**P1: Computer/Photogate Setup**

1. The *ScienceWorkshop* interface and the computer should be already switched on.

2. Connect the Photogate’s phone plug to Digital Channel 1 of the interface.

3. With the three-legged base ring stand and suitable rods and clamps, suspend the U-shaped photogate above the path the ball takes just after being launched. The ball should pass through the photogate, blocking and un-blocking the light beam of the photogate along the diameter of the ball as it passes without hitting the photogate. Be careful not to change this vertical position of the photogate after the initial setup.

4. Launch *DataStudio* and open the activity under PHYS_2211L/Ballistic Pendulum. Double-click on document titled as shown:

<table>
<thead>
<tr>
<th>DataStudio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed Through Gate.ds</td>
</tr>
</tbody>
</table>

5. The *DataStudio* document should open with a display of a window that indicates the *Speed* of the ball measured by the photogate, and another “*Calculator*” window. On this Calculator, change the value of “*x*” in cm given on the left side of the window to the measured diameter of the ball, if different and click on “*Accept*”. The photogate can now be activated to collect data by clicking the “*Start/Stop*” at the top of the computer screen.
P2: Equipment Setup and Data Acquisition

1. Determine the average mass and diameter of the metal ball using the appropriate instrument provided. Record these measurements in the appropriate spaces on the data sheet.

2. If the pendulum is still attached, loosen the thumbscrew holding the axis of rotation of the pendulum and carefully remove the pendulum from its support. Place the pendulum arm on the triple-beam balance and measure its mass. Record the average mass of the pendulum.

3. Prepare the gun for firing. Place the ball at the end of the firing rod and pushing it back on the gun, compressing the spring until the trigger is engaged. This process compresses the spring by a specific amount each time and thus gives the ball the same initial velocity every time the gun is fired.

   SAFETY REMINDER
   • DO NOT let the ball hit anyone. Place a suitable backstop to “catch” the ball. Announce your intention to fire and make sure that you have secured a clear firing zone BEFORE firing the gun.
   • Follow directions for using equipment.

4. We will directly measure the initial velocity of the ball using the Photogate assembly. The pendulum is not used in this step. Place the Photogate so as to record the motion of the ball just after firing, as described in Step #3 of Procedure P1. Place an appropriate backstop to prevent injury from the ball after firing. Pull the trigger.

5. Repeat this procedure four more times to obtain five readings. Record the readings of the measured velocity of the ball, as recorded by DataStudio.

6. Remove the Photogate assembly. Place the pendulum arm into its point of support and secure it with the thumbscrew so that it hangs and swings freely in the vertical position in front of the gun.

7. With the pendulum at rest in the vertical position, pull the trigger to fire the ball into the pendulum bob. This will cause the pendulum bob with the embedded ball to
rise to a certain height above the base of the apparatus. The pawl on the pendulum bob will engage a particular notch on the curved scale of the rack. Record the value on the scale at that notch position on the rack.

8. Repeat the Step #7 four more times, recording the position of the pendulum on the rack each time. Compute the average value of these positions and record this value on your data sheet.

9. Set the pendulum with its pawl engaged in the notch that corresponds most closely to the average value of the position calculated in Step 8 above. Measure the vertical height $h_1$ from the base of the apparatus to the index point attached to the pendulum. Read this measurement to 0.1 mm.

10. Let the pendulum hang freely at rest in its lowest position. Measure the vertical distance $h_2$ from the base of the apparatus to the index point. Read this measurement to 0.1 mm.

11. Calculate the initial velocity of the ball.
Lab Report Section

DATA ____________________________

1. Diameter of the ball = __________ cm
2. Mass of the ball, $m_b =$ __________ kg
3. Mass of the pendulum bob, $M_p =$ __________ kg
4. Highest notch reached by the ballistic pendulum on the curved scale of the rack:

Data Table #1:

<table>
<thead>
<tr>
<th>Trial</th>
<th>Measured velocity (m/s)</th>
<th>Deviation (m/s)</th>
<th>Curved scale reading</th>
<th>Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run #1</td>
<td></td>
<td></td>
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<tr>
<td>Run #2</td>
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<tr>
<td>Run #3</td>
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<tr>
<td>Run #4</td>
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<tr>
<td>Run #5</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Averages:</td>
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</tr>
</tbody>
</table>

5. Vertical height, $h_1 =$ __________ m
6. Vertical height, $h_2 =$ __________ m
7. Vertical height, $h = (h_1 - h_2) =$ __________ m
8. Calculated velocity of the pendulum and ball just after the collision,
   $V_f =$ __________ m/s
9. Calculated velocity of the ball before the collision,
   $v_b =$ __________ m/s
10. Average measured velocity from Data Table #1 above,
    $v_b'$ =$ __________ m/s
11. Percent difference between the values of the velocity before the collision,
    Percent difference = __________
CALCULATIONS ____________________________

1. Calculate the average value of the measured velocity for the five shots of the pendulum. Calculate the deviation of each measured velocity from the average and then calculate the average deviation (a.d.). Enter your result on line #10 of the Data Sheet. Attach the error to the measured velocity as: \textit{average velocity} \pm \textit{the a.d.}

2. Compute the average curved scale reading and the average deviation (a.d.) for the five shots of the pendulum. Enter this average in the appropriate space in the data table.

3. From the data of Procedure Steps #9 & #10, compute the vertical distance \( h \) through which the center of gravity of the loaded pendulum was raised and attach the error to this height by using the a.d. found in Calculation #2 above.

4. Calculate the velocity of the pendulum with the embedded ball just after the collision. Enter your answer on the data sheet.

5. Calculate the velocity of the ball before the collision using the measured values of the masses of the ball and the pendulum bob. Enter your answer on the data sheet.

6. Assume the average value of the velocity measured directly to be the accurate velocity. Compare the \textit{calculated} value of the velocity of the ball before the collision from Calculation #5 above with the \textit{measured} average value obtained directly in Calculation #2. Find the percent difference between these two values of the velocity.

7. Does the value of the calculated velocity from Calculation #5 fall within the range of error found in Calculation #1 for the velocity from direct measurement? What factors and reading errors directly influence the velocity values?
QUESTIONS

1. Show the derivation of the equation used in calculating the velocity of the ball before the collision.

2. Using the measured masses of the ball and pendulum from your experiment, calculate the kinetic energy of the ball just before the collision using the measured value of the velocity of the ball.

3. Calculate the kinetic energy of the pendulum bob and ball just after the collision from the calculated value of the velocity of the pendulum and ball.

4. (a) From the results of Calculations #2 and #3, calculate the change in the kinetic energy of the ball immediately before and after the collision as a fraction of the total energy, \( \frac{KE_f - KE_i}{KE_i} \).

   (b) Calculate the ratio of the mass of the pendulum bob to the total mass of the bob and the ball, \( \frac{M_p}{m_b + M_p} \).

5. (a) Using only equations, show that the fractional energy loss during the perfectly inelastic collision is equal to the ratio of the mass of the pendulum bob to the total mass of the bob and the ball. (b) Do your results in Calculation #4 above confirm this relationship?